Elimination Finals

1st Annual WSMA Math Bowl

May 27, 2011

Problem 1

What is the area of the region in the coordinate plane described by $|x + y| \leq 4$ and $|x - y| \leq 4$ that does not include the region described by $|x + y| \leq 2$?

Problem 2

Find the value or values of x that satisfy the following inequality:

$$x^2 - x + 1 \leq x^2 + x + 17 \leq 2x^2 + x + 1$$

Problem 3

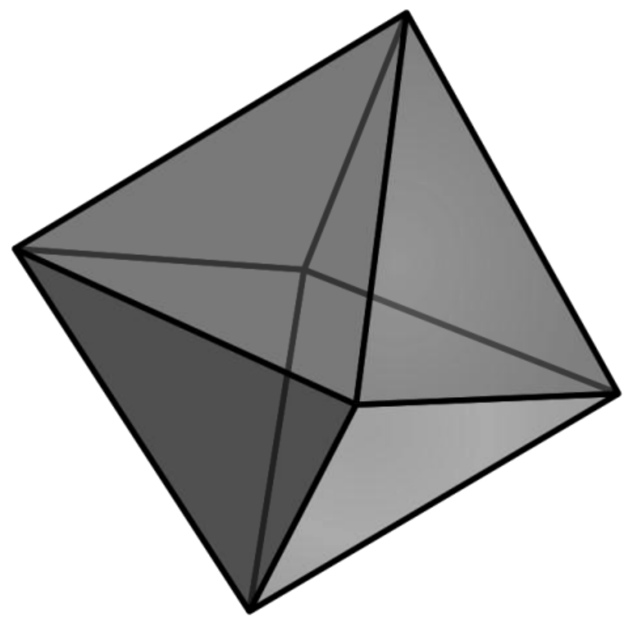
If $\mathrm{r} = \frac{x}{y}$ and $r^2 + 2r - 1 = 0$, express $y^4 - 12x^3y$ in terms of x only.

Problem 4

The parabolas $y = x^2 - 4x + 4$ and $y = -x^2 + 4x + 4$ intersect at points A and B. Similarly, the curves $y = x^2 + 8x + 5$ and $y = -x^2 - 8x - 9$ intersect at points C and D. What is the area of the trapezoid formed by the four points A, B, C, and D?

Problem 5

Find the volume of a regular octahedron with side length 2.



Problem 6

Find the sum of the following infinite series:

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} \cdots$$

Problem 7

Compute the absolute value of the difference between the repeating decimal $0.45454545\ldots$ and 0.45.

Problem 8

Find the period of

$$f(x) = \sin 5x + \cos 19x.$$

Problem 9

What is the smallest possible area of a right triangle whose side lengths are all composite numbers?

Problem 10

What is the third-smallest positive integer with exactly eight positive integer factors?

Problem 11

How many pairs of prime numbers less than 100 are there such that the sum of the two numbers in the pair is another prime number?

Problem 12

Suppose $5a^2 + 8ab + 5b^2 = 6$ and $3a + 3b = 4$. What is the value of $2a^2 + 5ab + 2b^2$?

Problem 13

Suppose the world is a cube, and the furthest distance between two cities on the same face is 20 units. What is the minimum volume of the world, in units cubed?

Problem 14

The sides of a trapezoid have lengths x, x, x, and $2x$. What is the measure of the greatest angle in the trapezoid, in degrees?

Problem 15

Quadrilateral $ABCD$ has midpoints M on AB, N on BC, O on CD, and P on DA. If M is located at $(0,0)$, N at $(5,0)$, and P at $(0,5)$, what are the coordinates of point O?

Problem 16

The product of R and 7 more than R is -12. What is the sum of the reciprocals of all possible values of R?

Problem 17

52 cards in a standard deck are dealt out randomly to the 25 Mariners and Ichiro's dog such that each is given a pair of cards. What is the probability that Felix Hernandez is dealt both a king and an ace?

Problem 18

Each vertex of a pentagon is assigned a nonnegative integer value so that the positive difference between the values at any two adjacent vertices is no greater than 2. How many ways are there to assign these five values if the sum of the values is 5? Two assignments are considered the same if one can be rotated to produce the other.

Problem 19

A cylindrical container 16 units high and 6 units in diameter is partially filled with water. The cylinder is tilted so that the water level reaches 11 units up the side of the cylinder at the highest but only 5 units up at the lowest. Compute the volume of water in the cylinder.

Problem 20

If $x^4 - 4^{x^3} + 6x^2 - 4x + 1 = \frac{4}{3}\log_2 4096$, compute the greatest positive real value of x.

Problem 21

Let N be an integer such that the product $41 \times 43 \times N$ can be written as the sum of 6 consecutive positive integers. Compute the least possible value for the smallest of the 6 integers.

Problem 22

Compute the ratio of the volume of a sphere to the volume of the largest regular octahedron that will fit inside it.

Problem 23

What is the greatest possible diameter of a circle centered at $(0, 0)$ that does not intersect the graph of $y = \frac{8}{x}$ at more than two points?

Problem 24

Evaluate $\frac{1}{\sqrt{11}+\sqrt{10}}+\frac{1}{\sqrt{10}+\sqrt{9}}+\frac{1}{\sqrt{9}+2\sqrt{2}}+\cdots+\frac{1}{\sqrt{2}+1}$.

Problem 25

The bases of a trapezoid have lengths 10 and 21, and the legs have lengths $\sqrt{34}$ and $3\sqrt{5}$. What is the area of the trapezoid? Express your answer as a fraction in simplest form.

Problem 26

Evaluate the sum: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} \cdots$

Problem 27

Find the volume of the solid formed by revolving an ellipse with semimajor axis of length 3 and semiminor axis of length 2 about its semimajor axis.

Well Done!

Thank you for participating!