Algebra Expressions

Why study *algebra*? Because this topic provides the mathematical tools for any problem more complicated than just combining some given numbers together.

Algebra lets you solve word problems in a regular and systematic way. Algebra lets you use symbols to solve all possible instances of a certain equation, not just a single example of the equation with certain numbers in it. For example, it will help you understand the properties of the universal equation for all straight lines (coming soon in lesson 15).

Really, What is Algebra?

Practically speaking, *algebra* has two main differences from the arithmetic you've studied so far:

1. We use variables as placeholders for unknown quantities.

2. We work on whole equations rather than just numbers.

What we're teaching in algebra is "how you can torture equations" to get the information you want. We will happen to do some arithmetic along the way, but our focus is shifting away from manipulating numbers and toward changing equations. Sometimes the information you want is a simple number for a variable (like a number of inches), and sometimes you want to know how one variable is related to another (like Fahrenheight is related to Celsius temperature).

Using Variables

It is common to have problems in which something is unknown. For example:

$3 \times 4 =$ _____ or $7 +$ _____ $= 10$

In *algebra*, we change from using blanks or question marks to using letters or variables. (Of course, the blanks above may be correctly filled in using 12 and 3, respectively!) Using '$\boldsymbol{x}$' to stand for the first blank number and '$\boldsymbol{y}$' for the second we could write:

$3 \times 4 = \boldsymbol{x}$ and $7 + \boldsymbol{y} = 10$

These are really the same problems as the fill-in-the-blank problems above. In this case, we could say the solution is $x = 12$ in the first equation and $y = 3$ in the second.

The letters are *variables* that are really just unknown numbers as was the blank. We can use variables in any way that we could use other numbers. We can add and subtract, multiply and even use them in powers and square roots.

There isn't anything special about *'x'* and *'y'*, either. Although they are the most common letters used, we can choose any letters we want. We will often choose letters that remind us of the missing number. For instance, we might use *h* for height, *l* for length, *v* for volume and *t* for time.

Letters are not reserved, and you really can choose what ever you like. But there are some commonly used ones. The letters *a*, *b* and *c* are often used to indicate a constant. They indicate a number of unknown values that are not variable; they are unchanging numbers. We just don't know what numbers they are.

A Short Form for Multiplication

Wait a minute! There could be some confusion with the letter *x*. Does it mean the unknown number '**x**' or to multiply? To avoid confusion, we have some new ways to indicate multiplication.

We can write the two multiplicands next to each other. If multiplication is not obvious, we can use a dot • as a separator.

In fact, we will rarely use the symbol "×" anymore:

- Use a dot • between two numbers: 4•7 *"four times seven"*
- Or just write a number by a variable: $8t$ *"eight times t"*
- Write a value next to parentheses: $4(x+1)$ *"four times x plus one"*
  or: 2(3) *"two times three"*
- Write a value next to an operation: $4\sqrt{25}$ *"4 times the square root of 25"*
- Write to operations next to each other: 3!4! *"3 factorial times 4 factorial"*
- Write parentheses next to each other: $(x+1)(x-1)$ *"x plus 1 times x minus 1"*

Neatness counts! You must make the dot • clearly different than a decimal point. Be sure the reader can tell when you mean 4•7 instead of 4.7!

Expressions

Expressions are numbers combined together using various arithmetic operations.

For example, the expression 4 • 9 / 6 combines the numbers 4, 9 and 6 by multiplying 4 by 9 and then dividing by 6 giving the result 6.

Sometimes an expression will involve a variable. You can simplify the expression by substituting the value of the variable into the expression and compute the result:

Example: Find **3*x* - 6** if you are given that ***x* = 7**

Solution:	Put the value of 7 in for *x*	3 • 7 - 6
	Multiply by 3 to get 21	21 - 6
	Subtract 6 to get 15	15

Equations

An *equation* is a statement in which one expression is equal to another.

Examples: $24 - x + 6 = 17$ or $4y + 2 = 8y - 3$

We can use equations to give us information about unknown numbers as we did in the fill-in equations at the top. We call this *solving for a variable*. If we are asked to solve for *x* in the equation:

$$5x = 20$$

Then we would answer $x = 4$. (Remember *x* in this case is just an unknown number, not multiply).

Now think about what you just did to solve for *x*. You probably divided 20 by 5. You reversed the arithmetic shown in the equation. How does algebra help you do this work? You divide both sides of the equation by the same amount, choosing a value to simplify the arithmetic. Let's divide both sides by 5:

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = \frac{20}{5} = 4$$

Think of the equals sign '=' as a balance. You can modify one side however you want, as long as you change the other side in the exact same way to keep it in balance:

- You can multiply (or divide) both sides by the same number or expression.
- You can add (or subtract) both sides by the same number or expression.
- You can even square (or use an exponent or square root) on both sides.
- You can add (or multiply) a variable on both sides, as long as it is the same variable.
- You can swap sides across the equals sign: $a+b=x$ is the same as $x=a+b$.

The only thing you are not allowed to do is divide by zero.

Please note there is an implied parenthesis around each side of an equation. So whatever you do must apply to everything.

Example: Multiply

Solving for a Variable

A common use for algebra is solving for one variable in an equation. You solve it by applying the same operation to both sides of the equation, to simplify it until the variable is alone on one side and the answer is on the other.

Example: Solve for n: $9-n=5$

Solution: Add n to both sides: $9-n+n=5+n$
or: $9=5+n$

Subtract 5 from both sides: $9-5=5+n-5$
or: $4=n$

We prefer to write it so the variable is on the left: $n=4$

The key point to "solve for a variable" is to work backwards through the equation. The goal is to manipulate the equation to get the unknown variable by itself on one side of the equation. It takes some creativity and work to figure out how to simplify.

Example: Solve for x: $5x - 6 = 19$

Solution: Add 6 to both sides: $5x - 6 + 6 = 19 + 6$

or: $5x = 25$

Divide both sides by 5: $5x/5 = 25/5$

or: $x = 5$

Check: Put 5 into original problem: $5 \cdot 5 - 6 = 19$? *Yes!*

The variable itself can be operated upon, just like any other part of the equation. For example, the variable might begin at some unusual place. You can add, subtract, multiply and divide it like the other parts.

Example: Solve for x: $3 = \frac{54}{x}$

Solution: Multiply both sides by x: $3x = \frac{54}{x}x$

or: $3x = 54$

Finally, this looks more familiar.

Divide both sides by 3: $\frac{3x}{3} = \frac{54}{3}$

or: $x = \frac{54}{3} = 18$

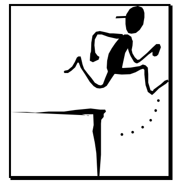
Note that using exponents (such as squaring) affects the whole expression on each side. You can't just, say, use the square root on a part of one side.

Example: Solve for x: $x^2 + 16 = 25$

Solution: Take the square root: $\sqrt{x^2 + 16} = \sqrt{25}$ ***Right!***

$\sqrt{x^2} + \sqrt{16} = \sqrt{25}$ ***Wrong!***

You will generally find the last operation is the first thing to undo. Imagine you are computing the equation on a calculator. The **last** operation you would type is likely to be the **first** operation to undo. Let's look at a really big and messy example; try to spot how we're working backwards to undo it!



Big Messy Example: Solve for *x*: $2(x-1)^2+3=21$

Solution: Subtract 3 from both sides: $2(x-1)^2+3-3=21-3$

or: $2(x-1)^2=18$

Divide both sides by 2: $\frac{2(x-1)^2}{2}=\frac{18}{2}$

or: $(x-1)^2=9$

Square root of both sides: $\sqrt{(x-1)^2}=\sqrt{9}$

or: $x-1=3$

Finally add 1 to both sides: $x-1+1=3+1$

or: $x=4$

Check: Substitute 4 into the original equation:

$2 \cdot (4-1)^2 + 3 = 21$

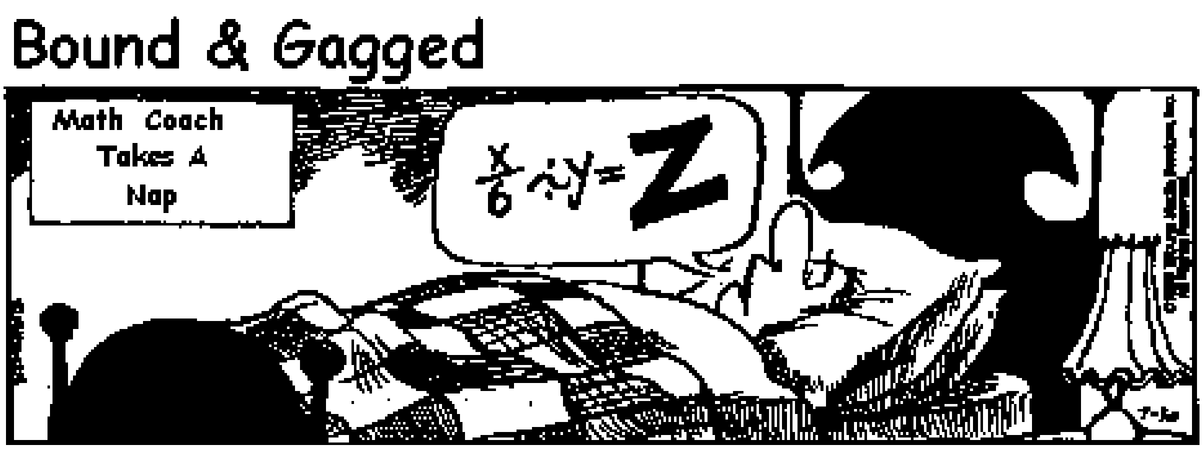
$2 \cdot 3^2 + 3 = 21$

$2 \cdot 9 + 3 = 21$

$21 = 21$ *Yes!*

Vocabulary

- *Algebra* - a branch of mathematics in which symbols are used to represent numbers or sets of numbers. This is not related to *algae,* a substance that grows on your homework if you drop it in Lake Sammamish.
- *Variable* - a symbol that takes the place of an unknown quantity. We can choose any letter, and usually choose letters that remind us of the missing value. For example, the equation for "rate times time equals distance" might be written as: $r \times t = d$
- *Expressions* - numbers and variables combined with arithmetic operations. For example, $(x-1)^2$ is an expression. It does not contain an equals sign.
- *Equation* - a statement that one expression is equal to another. For example, $(x-1)^2 = 9$ is an equation. This word is not related to *equator,* which is where you want to go during our rainy Seattle winter.



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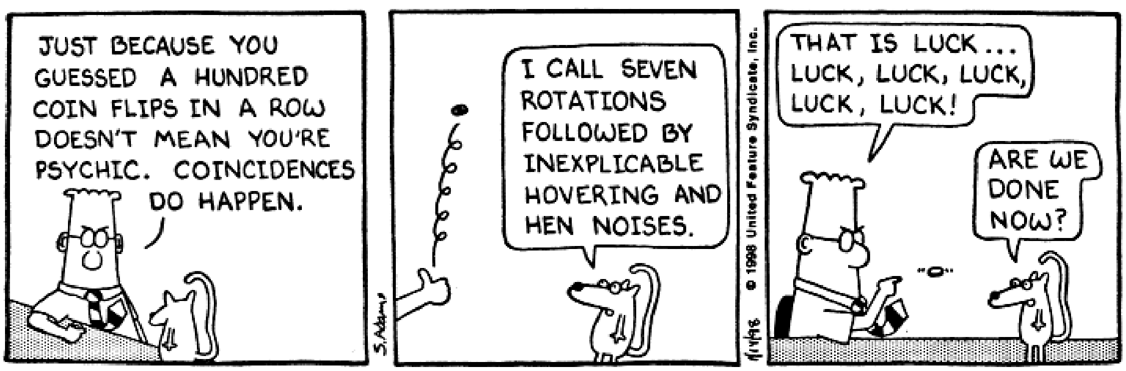

Algebra Expressions

1) Evaluate each expression when $\boldsymbol{a} = \mathbf{2}$, $\boldsymbol{b} = \mathbf{4}$, and $\boldsymbol{c} = \mathbf{10}$:

a) $1 + 3a$ *Example:* $1 + 3 \cdot 3 = 10$

b) $4b - a$ ____________

c) $c - b - a + 1$ ____________

d) $c - (b - a) + 1$ ____________

e) $ab - c$ ____________

f) $\dfrac{3c}{a+b}$ ____________

g) $\dfrac{c+a}{c-a}$ ____________

h) $\dfrac{b^2}{3!}$ ____________

i) $\dfrac{b \div a}{c+2}$ ____________

j) $\dfrac{2}{3}b + \dfrac{1}{2}c$ ____________

2) Solve for n in the following one-step equations. (Think about how you're solving them. Reverse the math to get n by itself.)

a) $2n = 10$ *Example:* $n = 10/2 = 5$

b) $n + 6 = 25$ $n =$ ____________

c) $9 - n = 5$ $n =$ ____________

d) $\frac{1}{6}n = 8$ $n =$ ____________

e) $n + \frac{1}{5} = \frac{7}{10}$ $n =$ ____________

3) Solve for n in these two-step equations. Always reduce fractions.
Hint: Check your work by substituting your answer into the original equation.

a) $2n + 1 = 2$ *Example:* $2n=1$, $n = ½$

b) $3n + 3 = 78$ $n =$ ____________

c) $6n - 3 = 1$ $n =$ ____________

d) $6 + 9n = 51$ $n =$ ____________

e) $\frac{n}{2} + 2 = 16$ $n =$ ____________

f) $\frac{n}{3} - 6 = 8$ $n =$ ____________

4) Solve for n in these two-step equations with parentheses.

a) $3(n-1)=9$ *Example:* $(n - 1) = 3$, $n = 4$

b) $5(n+4)=35$ $n =$ ____________

c) $6(n+4)=24$ $n =$ ____________

d) $2(n+2+3)=4$ $n =$ ____________

e) $12(n-2)=144$ $n =$ ____________

f) $\frac{1}{3}(n-5)=2$ $n =$ ____________

5) Solve for x in these one-step equations. Note that a, b, and c are constants. We don't know their value, we only know they represent some constant number.

a) $\frac{1}{a}x=8$ *Example:* $a\cdot\frac{1}{a}\cdot x=8\cdot a$, $x = 8a$

b) $\frac{1}{3}x=a$ $x =$ ____________

c) $x-b=5$ $x =$ ____________

d) $\frac{x}{a}=c$ $x =$ ____________

e) $ax=b$ $x =$ ____________

6) Solve for x in these equations, where x is on both sides.

a) $5x-2=2x+1$ *Example: 5x = 2x + 3, 3x = 3, x = 1*

b) $4x-1=x+2$ $x =$ ____________

c) $10x+2=x-43$ $x =$ ____________

d) $10x+5x+1=7x+49$ $x =$ ____________

7) Find a value of x (other than zero!) for these equations.
Hint: You may need to multiply (or divide) both sides by x.

a) $3x^2=6x$ *Example:* Divide both sides by x: $3x = 6$, so $x = 2$.

b) $2x^2=24x$ $x =$ ____________

c) $\frac{c}{x}=2$ $x =$ ____________

d) $9x^2-3x=0$ $x =$ ____________

e) $\frac{-4}{x}=22$ $x =$ ____________

f) $5=\frac{12}{x}$ $x =$ ____________

8) Mental Math: do these in your head, and write down the answers.
Leave all answers as reduced fractions, and in terms of radicals and pi.
This is very short to give you more time for the other algebra homework.

a) What is your name?

b) What is \$7.14 minus \$1.89?

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