Sometimes you need to know the probability of an "event" which is really the combination of various actions. It may be several dice rolls, or several cards selected from a deck, or a combination of different outcomes for different players.

How do you combine probabilities from different events?

Compound Probability

The combined likelihood of multiple events is called **compound probability**.

Suppose you have a weighted coin, that will show heads 75% of the time. If you flip it twice, do you have a 150% chance of getting two heads? Nope!

Probability is a fraction in the range of 0 to 1, or a percentage between 0 and 100%. When combining the probabilities from various events, it must give an answer in this range.

There are two ways to combine outcomes: OR and AND.

"**OR**" An example of "OR" probability is "What is the probability of drawing a 7 or a heart from a deck of cards?"
For one event OR another event, it increases the likelihood so **ADD** their probabilities. Be sure your events are not overlapping, or you may incorrectly count a favorable outcome twice.

"**AND**" An example of "AND" probability is "What is the probability of rolling a 3 and spinning a spinner onto the color blue?"
For one event AND then another, it decreases the likelihood so you **MULTIPLY** their probabilities. Be sure your second event is calculated assuming the first event is favorable.

Examples with Cards

Example: Suppose you draw two cards from a full deck of freshly shuffled cards. What is the probability of drawing a diamond on both cards?

Solution: Is this an OR or AND problem? Look at the wording. You can reword the statement as "what is the probability of drawing a diamond AND then drawing another diamond?" This is "AND" so plan on multiplying the separate probabilities from each card.

First draw: Choose from 13 diamonds out of 52 cards.

$$P(\text{diamond \#1}) = \frac{13}{52} = \frac{1}{4}$$

Second draw: Assume the first card was a favorable outcome (a diamond). Now there are only 12 diamonds and 51 cards remaining.

$$\text{P(diamond \#2)} = \frac{12}{51}$$

The problem asked for one event AND another, so multiply their probabilities: $\text{P(two diamonds)} = \frac{1}{4} \times \frac{12}{51} = \frac{3}{51}$ or about 5.9%

Examples with Coins

Example: Suppose you flip a coin three times. What is the probability of getting two heads and a tails, in any order?

Solution: We saw this problem in the last lesson, and solved it by listing all the outcomes and counting the desired outcomes. This time, let's solve it using compound probability.
Is this an OR or AND probability? Notice the problem can be reworded as "what is the probability of getting heads/heads/tails OR heads/tails/heads OR tails/heads/heads". So this is an "OR" type of problem, so we'll plan on adding the separate outcomes together.

$$\text{P(heads/heads/tails)} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$$

$$\text{P(heads/tails/heads)} = \frac{1}{8}$$

$$\text{P(tails/heads/heads)} = \frac{1}{8}$$

Adding these probabilities together results in the same answer as last week:

$$\text{P(two heads and a tails)} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

Example of Manufacturing

Example: The Pedantic Pet Porpoise Polisher produces a product called the "pet polisher". Each pet polisher consists of ten precision parts, which are produced separately and then assembled into one polisher. The PPPP (or as they like to call themselves, P^4) want to predict what percentage of finished polishers will work successfully after assembly.

Suppose P^4 knows that each part of a polisher has a 98% probability of being produced properly and performing perfectly. When they assemble ten of these parts together, what is the compound probability the finished pet polisher will work perfectly?

Solution: The problem is to find the probability that part #1 AND part #2 AND part #3 and so forth will all work successfully.

To solve, convert the percentage (98%) to a decimal number (0.98), and multiply the probabilities together:

P(all ten parts work) = $0.98 \times 0.98 \times \ldots \times 0.98$

P(all ten parts work) = $(0.98)^{10} = 0.817 = 81.7\%$

A success rate of 81.7% is not very good for a mass-produced gadget, so they will want to improve the quality of the individual parts that go into it.

Examples with Dice

Example: What is the probability of rolling a 3 or higher on a 6-sided die? (Note, one is "die" and two is "dice".)

Solution: The favorable outcomes are 3, 4, 5, or 6. That is, 4 in all. There are six possible outcomes.

$$\text{P(rolling a 3 or higher)} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

Types of Selection

Selection problems are of two flavors. The objects selected are either permanently removed, or returned to the pile where they can be selected again.

- *Selection with replacement* – An item is chosen at random, then returned to the pool of objects before drawing the next item. For example, choosing a playing card and returning it to the deck before choosing another random card. Also, rolling a die is equivalent to selection with replacement because the next random number can always include those numbers already rolled. In these problems, the number of possible outcomes remains the same for each selection.

- *Selection without replacement* – An item is chosen at random, and is not available to be chosen again in the next selection. Some examples would be drawing names from a hat, or dealing from a deck of cards, or taking socks from a drawer in the dark. In these problems, the number of possible outcomes is smaller after each selection.

In this sense, all probability problems can be thought of as "selection" problems. You always need to be aware of the number of possible of outcomes in compound probability problems.

Examples of Selection

Example: Mr. Hansen is removing random socks from his dresser in the dark. The drawer contains 3 black, 4 red, and 5 striped socks. What is the probability of taking two striped socks?

Solution: Is this an OR or AND probability? Notice it depends on drawing a striped sock AND THEN another striped sock. So it is AND and we will plan on multiplying the separate probabilities together.

There are 5 striped socks, and 12 socks altogether.

$$P(\text{striped sock \#1}) = \frac{\text{favorable outcomes}}{\text{possible outcomes}}$$

$$P(\text{striped sock \#1}) = \frac{5}{12}$$

For the second sock, we assume the first sock selection was successful. Now we can choose any of 4 striped socks out of 11 socks remaining.

$$P(\text{striped sock \#2}) = \frac{4}{11}$$

Now multiply the probabilities because it is an AND problem.

$$P(\text{two striped socks}) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$$

$$P(\text{two striped socks}) = 15.1\%$$

Example: An old bag of Halloween candy has 10 chocolate-covered broccoli stalks, and 2 delicious Twix bars. (And you hate green tree-like vegetables.)

a) What is the probability of pulling out two Twix bars at random, if you put the first one back in the bag before pulling out the second one?

b) What is the probability if the first bar is *not* replaced before choosing the second item?

Solution: a) The probability of getting a Twix on the first try is 2 out of 12, which equals 1/6. Assuming the first try is successful, the bar is replaced, and the second bar is drawn. The probability is the same for the second try.

$$P(\text{two Twix}) = P(\text{Twix \#1}) \times P(\text{Twix \#2})$$

$$P(\text{two Twix}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} = 2.78\%$$

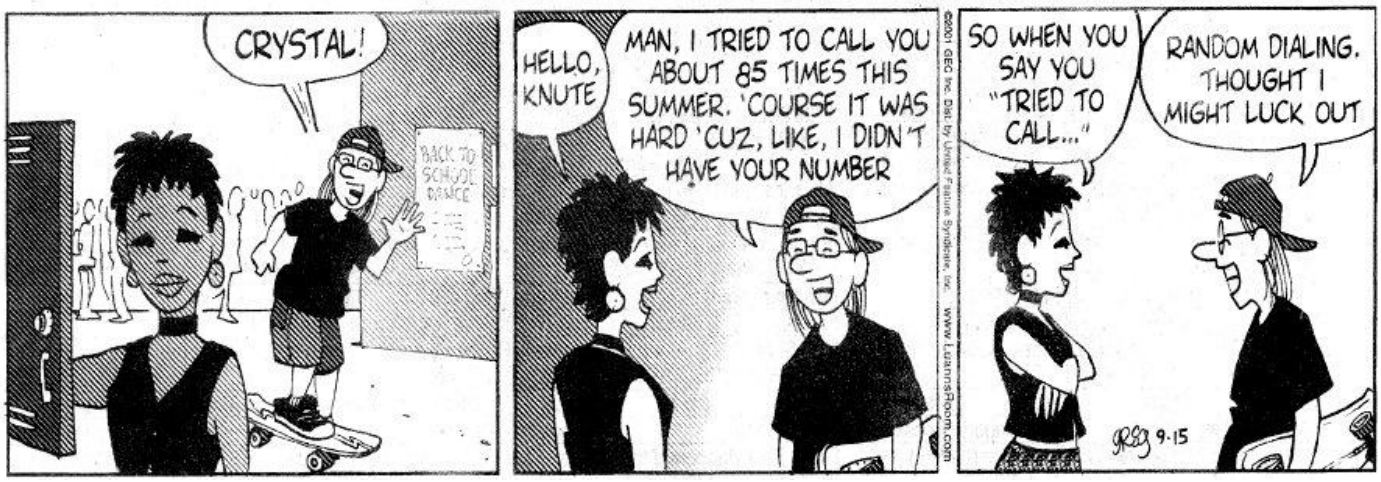
c) The probability of getting a Twix on the first try is 1/6, as above. Assuming the first try is successful, the second bar is drawn. Now there is one less Twix bar to choose from, and only 11 items in the bag.

$$P(\text{two Twix}) = P(\text{Twix \#1}) \times P(\text{Twix \#2})$$

$$P(\text{two Twix}) = \frac{1}{6} \times \frac{1}{11} = \frac{1}{66} = 1.51\%$$

Dilbert, by Scott Adams

Luann, by Greg Evans



Probably Homework

Find the compound probability of these events. *Hint:* Follow similar steps as in the previous Lesson 9, and count the favorable outcomes carefully.

1) You tried to call your friend by dialing telephone numbers at random. You know their phone number begins with 425-392-xxxx but you don't know the last four digits. What is the probability of reaching your friend on the first try?

2) You have an unfair coin that comes up heads 87% of the time.
   a) What is the probability of getting heads on both of 2 flips?

   b) What is the probability of getting heads on all of 3 flips?

   c) How many flips until the probability of getting all heads is practically 50%?

3) Answer these questions about compound probability using AND.
   a) Suppose your birthday is on Dec 6th, on which it rains 3 out of 4 years. What is the probability of a sunny birthday every year for the next three years?

   b) What is the probability of pulling the Ace of Spades at random from a deck of 52 cards, and then flipping a coin and it comes up tails?

c) What is the probability of drawing the Queen of Hearts at random from a deck of 52 cards, and then drawing another heart?
Hint: Remember to assume the first draw is successful when considering the second draw. That means one of the hearts is already removed.

d) Suppose the probability of rain is 75%. Furthermore, suppose your teacher assigns homework randomly just ½ of the time.
What is the probability you will be given homework on a rainy day?

4) A bag of marbles has 10 red and 6 blue marbles.
   a) What is the probability of pulling out 2 blue marbles if you put the first marble back in the bag before pulling out the second one?

   b) What is the probability of pulling 2 blue marbles out of the bag, if you don't put the first marble back in the bag before pulling out the second one? (*Hint:* The probability should become smaller than above.)

5) Here's a quiz about taking quizzes! You are a very good math student. You understand all the material and always check your work. However, nobody is perfect – so let's suppose accidents (mistakes) happen to you a mere 2% of the time. That is, on any given problem you have a 98% chance of answering it correctly.

   Mr. Hansen gives you a quiz with 8 problems. What is your probability of answering all eight correctly?

6) You have been captured by the nasty Pickled Porpoise and put in his dungeon. There are 3 doors leading out of the dungeon. You choose a door at random. Two doors lead to certain death, but one door leads down a passageway where a sleeping dragon lies. Your chances of tiptoeing past the sleeping dragon are 70% (he is a heavy sleeper). If you get past the sleeping dragon (he eats you if you wake him up) there is a pool of sharks must swim across. Your chances of swimming through the shark pit (and living) are ¾ (the sharks are not very hungry). If you make through the pit, you are free! What is the probability you will escape the evil Pickled Porpoise?

7) To protect yourself from Dr. Porpoise, you installed a computer-controlled high power coilgun protection system in your bedroom. (http://www.oz.net/~coilgun) There is a 10% chance your mother won't recognize him and will accidentally allow him to your room for a visit. The security code to safely enter your bedroom has three digits. If he guesses the wrong code, he dies. What is the probability the nefarious sea mammal will pass into your bedroom and live?

8) *Extra credit:* Suppose your sock drawer contains five red socks and five blue socks, and you choose some at random. *How many socks* must you choose to be certain that you will get a matching pair?

9) Mental Math: do these in your head, and write down the answers. When you're done *check your answer* with pencil and paper, or calculator, or a friend.

a) What is four-thirds of 33?

b) What color is an orange? ☺

c) What is –12 times 11?

d) What is your name?

e) What is $(-3)^3$?

f) Round the number 4.5999 to the nearest whole number, then multiply by 2.

g) What is 15 – (-8)?

h) What are the next numbers: 625, 125, 25, 5, ____, ____, ____

i) Did you check your answers? If not, go back and check your answers now. *Hint*: It's okay to ask someone to check your answer to 7b and 7d for you. And to check the other answers, too.

You're done! Detach the homework from the lesson, and turn in just the *homework*.